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THE EFFECTS OF THERMAL POLLUTION ON RIVER ICE CON-  
DITIONS

S. Lawrence Dingman, et al

Cold Regions Research and Engineering Laboratory  
Hanover, New Hampshire

December 1967

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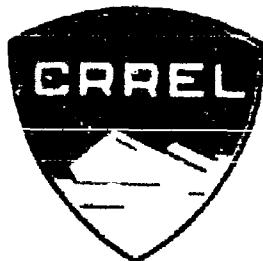
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**Research Report 206**

**THE EFFECTS OF THERMAL POLLUTION  
ON RIVER ICE CONDITIONS**

**I. A GENERAL METHOD OF CALCULATION**

by

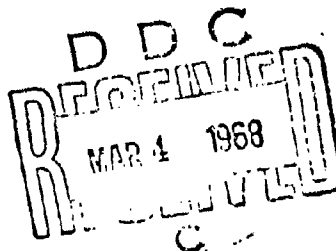
S.L. Dingman, W.F. Weeks

and

Y.C. Yen

DECEMBER 1967

U.S. ARMY MATERIEL COMMAND  
COLD REGIONS RESEARCH & ENGINEERING LABORATORY  
HANOVER, NEW HAMPSHIRE



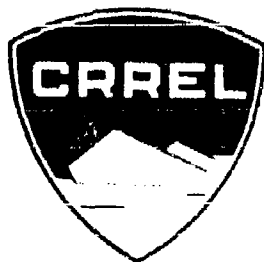
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## PREFACE

This paper was prepared by S. L. Dingman, hydrologist, Dr. W. F. Weeks, glaciologist; and Dr. Y. C. Yen, chemical engineer, U.S. Army Cold Regions Research and Engineering Laboratory. Dr. Yen is Head, Physical Sciences Branch, Research Division.

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# CONTENTS

	Page
Preface .....	ii
Summary .....	iv
Introduction .....	1
Previous work .....	1
List of symbols .....	3
Basic equation .....	5
Evaluation of $Q^*$ .....	7
$Q_R$ .....	8
$Q_B$ .....	9
$Q_E + Q_H$ .....	10
$Q_S$ .....	13
$Q_G$ .....	13
$Q_{GW}$ .....	13
$Q_F$ .....	15
Calculation of $X$ .....	15
"Experimental" checks .....	16
Applications to the St. Lawrence Seaway .....	18
Conclusions .....	20
Literature cited .....	21
Appendix A: Computer program for calculating water temperatures below thermal pollution sites .....	23
Appendix B: Computer program for calculating heat budget components and total heat loss rates from a cooling river .....	29
Appendix C: Meteorological data, U.S. Weather Bureau, Minneapolis-St. Paul Airport and thermal pollution, $Q_p$ , from the Northern States Power Company Riverside and Highbridge plants and the Minneapolis-St. Paul Sanitary District plant .....	33

# ILLUSTRATIONS

Figure	
1. Definition sketch for basic equation .....	6
2. $(Q_E + Q_H)$ vs $(T_w - T_a)$ at $v_a = 0, 5$ and $10 \text{ m sec}^{-1}$ , $U = 100\%$ .....	12
3. $Q_B, Q_E, Q_H$ , and $Q^*$ vs $(T_w - T_a)$ at $v_a = 0, 5$ , and $10 \text{ m sec}^{-1}$ ; $U = 50\%$ ; clear-sky conditions .....	14
4. $Q_S$ vs $(T_w - T_a)$ at $v_a = 0$ .....	14
5. Typical downstream temperature profile below a thermal pollution source .....	15
6. Observed vs calculated ice-free reaches .....	18
7. Length of ice-free reach as a function of air temperature and wind speed, South Shore Canal, St. Lawrence Seaway .....	20

# TABLES

Table	
1. Observed and calculated values for the areas and lengths of the ice-free reaches below thermal pollution sites along the Mississippi River, Minneapolis-St. Paul, Minnesota .....	17

## SUMMARY

The problem of predicting downstream changes in river temperatures due to the addition of various forms of thermal pollution can be expected to assume greater importance as man increasingly uses river water as a coolant and as a medium for transporting biological wastes. A possible result of such pollution is the maintenance of ice-free river reaches during the winter which might have beneficial consequences to river traffic on navigable waterways. Previous methods for predicting downstream temperature decrease, which could be used to estimate lengths of ice-free reaches, are limited in that only a few of the several pertinent meteorological factors determining heat loss rates are considered.

This report formulates a differential equation for the steady-state heat balance of a volume element of a river, which leads to the expression

$$x = C_x \int_{T_{w0}}^{T_{wx}} \frac{dT_w}{Q^*}$$

where  $x$  is the distance downstream from the thermal pollution site to the cross section where water temperature equals  $T_{wx}$ ,  $T_{w0}$  is the water temperature at  $x$  equals zero,  $Q^*$  is the rate of heat loss from the water surface, and  $C_x$  is a constant which includes flow velocity and depth.  $Q^*$  is the sum of heat losses due to evaporation, convection, long- and short-wave radiation, and other processes, each of which is evaluated by an empirical or theoretical expression. For given hydrometric and meteorological conditions (air temperature, wind speed, humidity, cloud cover, cloud height, etc),  $1/Q^*$  is a complicated function of water temperature. A computer program numerically evaluates the integral and calculates the distance  $x$  for given values of  $T_{wx}$ . The value of  $x$  when  $T_{wx}$  equals  $0^\circ\text{C}$  is taken as the length of the ice-free reach.

There are two principal limitations to accurate prediction of downstream temperature changes. First, it is difficult to estimate the degree of lateral mixing in a natural river. Second, there is considerable uncertainty as to how to best evaluate convective and evaporative heat losses under conditions of local atmospheric instability. Two approaches are used in this report, both based on the application of the Bowen ratio to: (1) the widely used Kohler equation for evaporative heat loss; and (2) an equation for convective heat loss developed empirically by Russian workers under wintertime conditions. The latter equations predict quite well the observed lengths of ice-free reaches associated with power and sewage plants along the Mississippi River at Minneapolis - St. Paul, Minnesota.

It is shown that significant portions of the St. Lawrence Seaway can be kept ice-free by the installation of nuclear reactors at appropriate locations.

# THE EFFECTS OF THERMAL POLLUTION ON RIVER ICE CONDITIONS

## I. A GENERAL METHOD OF CALCULATION

by

S. L. Dingman, W. F. Weeks, and Y. C. Yen

### INTRODUCTION

The general problem of the magnitude and extent of downstream changes in river temperature due to the addition of "thermal pollution" in various forms is assuming greater importance as man increasingly uses river water as a coolant and as a medium for transporting biological wastes. This paper treats a special case: the effects of such heat additions on river ice conditions. The basic approach, however, is applicable to the general problem.

If temperature increases due to thermal pollution are of sufficient magnitude, an appreciable reduction in thickness, or even complete disappearance, of the winter ice cover will result. The thermonuclear production of electrical power with the associated production of large quantities of waste heat is now becoming competitive with the more conventional methods of power production. If it can be established that this waste heat is sufficient to keep major portions of navigable rivers free from ice, the proper positioning of the thermonuclear reactors might permit the effective use of the river by shipping throughout a significant portion of the winter. The advantages of being able to calculate the effects of changes in the amount of thermal pollution, the hydrometric parameters of the stream, and the weather conditions on the position of the ice front below a pollution site are therefore obvious.

### PREVIOUS WORK

While heat balance studies of lakes are numerous, very few such investigations exist for rivers. Eckel and Reuter (1950), who recognized the need for such work, were concerned primarily with predicting temperatures in rivers under natural conditions. Therefore they attempted to develop an equation to estimate the changes in temperature of a reach of river or moving parcel of water with time, rather than with distance. They considered heat budget items associated with short- and long-wave radiation and evaporation and convection to the atmosphere. Because of the complexity of the total heat balance term, the differential equation they developed could not be solved in closed form, and they used graphical iterative procedures which gave satisfactory agreement between measured and calculated river temperatures, at least for clear summer days.

Pruden *et al.* (1954) studied the heat balance of the St. Lawrence River in an attempt to determine the feasibility of keeping much of the river ice-free for parts of the winter. They considered the meteorological dependence of the various heat balance terms in somewhat more detail than had Eckel and Reuter. Because plots of the heat loss rate ( $Q^*$ ) vs air temperature ( $T_a$ ) and vs water temperature ( $T_w$ ) were close to linear (using average monthly values for meteorologic conditions), they wrote

$$Q^* = 183.6 + 15.5 T_a + 43.1 (T_w - T_a) \quad (1)$$



## 2 THE EFFECTS OF THERMAL POLLUTION ON RIVER ICE CONDITIONS

where  $Q^*$  is in  $\text{cal cm}^{-2} \text{ day}^{-1}$  and  $T_w$  and  $T_a$  are in degrees C. This expression for  $Q^*$  was then incorporated in a differential equation for cooling rate with distance, which is similar to the one derived in this paper. The inverse of their expression for  $Q^*$  can be integrated in closed form, and they found a simple exponential decrease of water temperature with distance which is very similar to that found in the present paper (Figure 5 shows a typical, nearly linear exponential curve). Their approach is, however, less satisfactory, as their expression for  $Q^*$  was developed for monthly average values at one general location only, and does not take into account the many meteorologic variables which affect the heat loss rate. Wind speed, for example, is shown here to be an extremely important factor in determining  $Q^*$ .

Ince and Ashe (1964) used observations from the St. Lawrence River to compare two methods of calculating downstream water temperature decreases. Both of these are based on a finite difference approach for a parcel of water moving at the average velocity of the river:

$$T_{w_2} - T_{w_1} = \frac{Q \uparrow}{D \rho_w C_p} \quad (2)$$

where  $Q \uparrow$  is heat-loss rate ( $\text{cal sec}^{-1}$ ),  $D$  is river discharge ( $\text{cm}^3 \text{ sec}^{-1}$ ),  $\rho_w$  is mass density of water ( $\text{g cm}^{-3}$ ),  $C_p$  is specific heat of water ( $\text{cal g}^{-1} \text{ } ^\circ\text{C}^{-1}$ ), and  $(T_{w_2} - T_{w_1})$  is temperature change of the water parcel over some time period ( $^\circ\text{C}$ ). In the first method, heat exchanges due to radiation, evaporation, convection and precipitation were accounted for in evaluating  $Q^*$ , using daily averages of the meteorological variables, but no details of the various expressions were presented. This approach was compared by Ince and Ashe (1964) to that using the simple formula

$$Q^* = 46.4 (T_w - T_a) \quad (3)$$

recommended by the Canadian Joint Board of Engineers, where  $Q^*$  is heat loss rate in  $\text{cal cm}^{-2} \text{ day}^{-1}$  and  $(T_w - T_a)$  is based on daily averages of air and water temperatures in  $^\circ\text{C}$ . Both approaches gave satisfactory estimates of water temperature under certain conditions.

While the basic approaches of these earlier studies are sound, and led to some success in river temperature prediction, they are all limited in some respect. The principal limitations lie in the formation of the expressions for heat loss rate  $Q^*$ . The present paper largely overcomes these limitations by use of formulas for the heat budget components that can be used with daily averages of meteorological variables and by use of a computer program which allows numerical integration of a more complete and complicated expression for  $1/Q^*$ . The limitations of the present approach arise from: 1) uncertainties in estimating the evaporative and convective components of heat loss from open water during periods of pronounced local atmospheric instability; 2) difficulties in evaluating the degree of mixing of the heated water through the stream cross-section; and 3) the assumption of steady conditions.

# THE EFFECTS OF THERMAL POLLUTION ON RIVER ICE CONDITIONS 5

## LIST OF SYMBOLS

When units used in the derivation in the text are different from those used in the computer program, the units used in the computer program are underlined.

Text	Meaning	Units	Fortran
A	Snow accumulation rate	$\text{g cm}^{-2} \text{ day}^{-1}$	ACC
C	Cloud cover	dimensionless	CLDC
$C_i$	Specific heat of ice	$\text{cal g}^{-1} \text{ }^\circ\text{C}^{-1}$	
$C_p$	Specific heat of water	$\text{cal g}^{-1} \text{ }^\circ\text{C}^{-1}$	CP
D	River discharge	$\text{cm}^3 \text{ sec}^{-1}$	
E	Evaporation rate	$\text{cm day}^{-1}$	
H	Cloud height	m	CLDH
J	Thermal equivalent	$\text{dy cm cal}^{-1}$	
L	Latent heat of fusion of ice	$\text{cal g}^{-1}$	
P	Atmospheric pressure	mb	PRES, P
$Q_a$	Heat added to river by long-wave radiation	$\text{cal cm}^{-2} \text{ day}^{-1}$	QA
$Q_{ar}$	Long-wave radiation reflected at river surface	$\text{cal cm}^{-2} \text{ day}^{-1}$	QAR
$Q_B$	Net heat lost from river as long-wave radiation	$\text{cal cm}^{-2} \text{ day}^{-1}$	QB
$Q_{bs}$	Long-wave back radiation from river surface	$\text{cal cm}^{-2} \text{ day}^{-1}$	QBS
$Q_{CL}$	Incoming clear sky short-wave radiation	$\text{cal cm}^{-2} \text{ day}^{-1}$	QCL
$Q_E$	Heat lost from river by evaporation	$\text{cal cm}^{-2} \text{ day}^{-1}$	QE
$Q_F$	Heat produced by fluid friction	$\text{cal cm}^{-2} \text{ day}^{-1}$	
$Q_G$	Heat added to river by geothermal heat flow	$\text{cal cm}^{-2} \text{ day}^{-1}$	
$Q_{GW}$	Heat added to river by ground-water inflow	$\text{cal cm}^{-2} \text{ day}^{-1}$	
$Q_H$	Heat lost from river by convection to atmosphere	$\text{cal cm}^{-2} \text{ day}^{-1}$	QH
$Q_P$	Heat added to river as thermal pollution	$\text{cal sec}^{-1}$	POLL
$Q_R$	Heat added to river by short-wave radiation	$\text{cal cm}^{-2} \text{ day}^{-1}$	QR

#### 4 THE EFFECTS OF THERMAL POLLUTION ON RIVER ICE CONDITIONS

##### LIST OF SYMBOLS (Cont'd)

Text	Meaning	Units	Fortran
$Q_{RI}$	Short-wave radiation incident at river surface	$\text{cal cm}^{-2} \text{ day}^{-1}$	QRI
$Q_{RR}$	Short-wave radiation reflected at river surface	$\text{cal cm}^{-2} \text{ day}^{-1}$	QRR
$Q_S$	Heat lost from river due to in-falling snow	$\text{cal cm}^{-2} \text{ day}^{-1}$	QS
$Q_1$	Heat advected into elemental volume	$\text{cal sec}^{-1}$	
$Q_2$	Heat conducted into elemental volume	$\text{cal sec}^{-1}$	
$Q_3$	Heat advected from elemental volume	$\text{cal sec}^{-1}$	
$Q_4$	Heat conducted from elemental volume	$\text{cal sec}^{-1}$	
$Q_l$	Heat-loss rate from water surface	$\text{cal sec}^{-1}$	
$Q^*$	Heat-loss rate per unit area of water surface	$\frac{\text{cal cm}^{-2} \text{ sec}^{-1}}{\text{cal cm}^{-2} \text{ day}^{-1}}$	QSTAR
R	Bowen ratio	dimensionless	
S	Slope of river surface	dimensionless	
$T_a$	Air temperature	$^{\circ}\text{K}, ^{\circ}\text{C}$	TA, TAC
$T_i$	Initial water temperature produced by thermal pollution	$^{\circ}\text{K}, ^{\circ}\text{C}$	TWIN, TWINC
$T_{nat}$	Natural water temperature above thermal pollution site	$^{\circ}\text{K}, ^{\circ}\text{C}$	TWNAT, TWNATC
$T_0$	Datum temperature	$^{\circ}\text{K}$	
$T_w$	Water temperature	$^{\circ}\text{K}, ^{\circ}\text{C}$	TW, TWC
$T_x$	Water temperature at distance x below thermal pollution site	$^{\circ}\text{K}, ^{\circ}\text{C}$	TWINIT, TWFINL
U	Relative humidity	percent	U
V	Visibility	km	V
X	Total length of ice-free reach	cm, m	
a	Empirical constant (eq 20)		A
b	Empirical constant (eq 20)		B
$e_a$	Vapor pressure of air	mb	EA
$e_{sa}$	Saturation vapor pressure of air at temperature $T_a$	mb	VAP(TA)

# THE EFFECTS OF THERMAL POLLUTION ON RIVER ICE CONDITIONS 5

## LIST OF SYMBOLS (Cont'd)

Text	Meaning	Units	Fortran
$e_{sw}$	Saturation vapor pressure of air at temperature $T_w$	mb	VAP(TW)
	Average depth of river	cm, <u>m</u>	H
k	Thermal conductivity of water	cal cm <sup>-1</sup> sec <sup>-1</sup> °C <sup>-1</sup>	
$v_a$	Wind velocity	m sec <sup>-1</sup>	VA
$v_w$	Average river flow velocity	cm sec <sup>-1</sup> , <u>m sec<sup>-1</sup></u>	VW
w	Average river width	cm, <u>m</u>	W
x	Distance downstream from thermal pollution site	cm, <u>m</u>	X
$\gamma$	Weight density of water	dycm <sup>-3</sup>	
$K$	Ratio $K_h/K_w$	dimensionless	FUG, K
$K_h$	Eddy diffusivity of heat in air	cm <sup>2</sup> sec <sup>-1</sup>	
$K_w$	Eddy diffusivity of water vapor in air	cm <sup>2</sup> sec <sup>-1</sup>	
$\rho_w$	Mass density of water	g cm <sup>-3</sup>	
$\sigma$	Stefan-Boltzmann constant	cal cm <sup>-2</sup> day <sup>-1</sup> °K <sup>-4</sup>	SBC

## BASIC EQUATION

The differential equation for downstream changes in water temperature under certain simplifying assumptions is developed as follows. Referring to Figure 1, consider a stream, carrying a steady flow, whose geometry is constant in space and time. Assume further that meteorological conditions above the stream are spatially and temporally constant. If water temperature changes in the downstream (x) direction only, the amount of heat advected into volume element  $w \cdot h \cdot dx$  in unit time,  $Q_1$  (cal sec<sup>-1</sup>), is

$$Q_1 = \rho_w v_w h w C_p (T_w - T_{nat}). \quad (4)$$

The heat transfer to the volume by conduction,  $Q_2$ , is

$$Q_2 = -k h w \frac{dT_w}{dx}. \quad (5)$$

Here  $\rho_w$  is the mass density of the water (g cm<sup>-3</sup>),  $v_w$  is the flow velocity of the water (cm sec<sup>-1</sup>),  $h$  is the flow depth (cm),  $w$  is the stream width (cm),  $C_p$  is the specific heat of the water (cal g<sup>-1</sup> °K<sup>-1</sup>), considered constant in the temperature range  $T_w$  to  $T_{nat}$ ,  $T_w$  is the water temperature (°K),  $k$  is the eddy

## 6 THE EFFECTS OF THERMAL POLLUTION ON RIVER ICE CONDITIONS

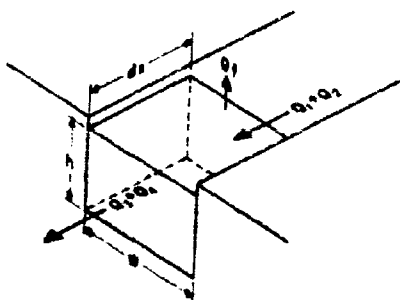


Figure 1. Definition sketch for basic equation.

thermal conductivity of the water ( $\text{cal cm}^{-1} \text{sec}^{-1} \text{ } ^\circ\text{K}^{-1}$ ), and  $T_{\text{nat}}$  is the natural water temperature just upstream from the thermal pollution site ( $^\circ\text{K}$ ). The amount of heat advected from this volume is

$$Q_3 = \rho_w v_w h w C_p \left[ (T_w - T_{\text{nat}}) + \frac{dT_w}{dx} dx \right] \quad (6)$$

and that leaving by conduction is

$$Q_4 = -khw \left[ \frac{dT_w}{dx} + \frac{d^2 T_w}{dx^2} dx \right]. \quad (7)$$

The water surface acts as a heat sink, and the total heat lost from it,  $Q^*$ , is

$$Q^* = Q^* \cdot w \cdot dx, \quad (8)$$

where  $Q^*$  is the heat loss rate per  $\text{cm}^2$  of water surface. A heat balance statement for the volume of water may be written as

$$Q_1 + Q_2 = Q_3 + Q_4 + Q^*, \quad (9)$$

which reduces to

$$Q^* = -\rho_w v_w h C_p \frac{dT_w}{dx} + kh \frac{d^2 T_w}{dx^2}. \quad (10)$$

Neglecting the second order (conduction) term, eq 10 simplifies to

$$x = -\rho_w v_w h C_p \int_{T_i}^{T_x} \frac{dT_w}{Q^*}, \quad (11)$$

where

$$T_i = T_{\text{nat}} + \frac{Q_p}{\rho_w v_w h w C_p}, \quad (12)$$

$T_x$  = water temperature at  $x$ .

$T_{\text{nat}}$  is the natural water temperature just upstream from the thermal pollution site, and  $Q_p$  is the heat of pollution ( $\text{cal sec}^{-1}$ ). By evaluating eq 11 at different values of  $T_x$ , the cooling curve downstream from the thermal pollution site can be obtained.

## THE EFFECTS OF THERMAL POLLUTION ON RIVER ICE CONDITIONS 7

Before examining  $Q^*$  further, the assumptions involved in the derivation of eq 11 should be considered. In most cases, the assumption of steady flow is reasonably good for ice-covered rivers, and, unless large tributaries enter, discharge can be considered nearly constant over fairly long reaches. Where large tributaries are present, or where there are significant changes in stream geometry along the reach of interest, the total reach can be subdivided and each subreach considered separately. A similar approach could be used to consider spatially varying meteorologic conditions.

The assumption that water temperature is constant throughout any cross section of the river is more difficult to evaluate. The intensity of turbulent mixing in a given direction equals the product of the coefficient of turbulent diffusion and the velocity gradient in that direction. Thus vertical mixing tends to be more complete (and hence temperatures more uniform vertically) in a shallow, fast stream than in a deep, slow one. By the same reasoning, one would expect narrow, fast streams to have more complete lateral mixing and more uniform temperature horizontally than wide, slow streams. Sayre and Chamberlain (1964) found that the coefficient of lateral turbulent diffusion is approximately proportional to the  $3/2$  power of the mean flow depth, so that one may add that, velocity gradients and other things equal, greater depth of flow promotes lateral mixing.

If there is a basis for estimating the lateral temperature gradient at various points along the reach of interest, consecutive sub reaches of varying width can be used to approximate the natural conditions. Where no such information exists, one must be satisfied with the assumption of complete isothermality in all cross sections. For an example of how erroneous this assumption can be, see Mackay (1966).

### EVALUATION OF $Q^*$

To solve eq 11, the heat transferred from the water to the air must be written as a function of water temperature  $T_w$ . To do this, a second heat balance equation is written in which all the modes by which heat can enter and leave the volume of water  $w \cdot h \cdot dx$ , aside from advection, are enumerated. The magnitude of the heat transferred by each mode is written as a function of hydrometric and meteorological variables, and the sum of these equated to  $Q^*$ . As shown below, the terms which contribute significantly to  $Q^*$  can be evaluated under given climatic and flow conditions.

$Q^*$  can thus be written as

$$Q^* = Q_R - Q_B - Q_E - Q_H - Q_S + Q_G + Q_{GW} + Q_F \quad (13)$$

where

- $Q_R$  is heat added at the river surface as short-wave radiation;
- $Q_B$  is net loss of heat by the exchange of long-wave radiation with the atmosphere;
- $Q_E$  is heat loss due to evaporation;

## 8 THE EFFECTS OF THERMAL POLLUTION ON RIVER ICE CONDITIONS

$Q_H$  is sensible heat lost by conduction and subsequent convection from the water to the atmosphere;

$Q_S$  is heat lost as a result of snow falling into the water;

$Q_G$  is heat added by the flow of geothermal heat through the stream bed;

$Q_{GW}$  is heat added by the flow of ground water into the stream through its bed and banks;

$Q_F$  is heat increase due to fluid friction.

With these definitions,  $Q^*$  is negative when there is a net loss from the volume  $w \cdot h \cdot dx$ .

### $Q_R$

Direct measurements of solar and atmospheric short-wave radiation at the site under consideration are obviously best for determining the magnitude of the energy supplied by this means. Where these do not exist, measurements from stations at some distance from the site can be substituted, provided there is reason to believe that the cloud cover conditions at the two sites are comparable during the period of concern. If direct measurements of short-wave radiation are lacking, estimates of its magnitude can be made by using tables, graphs, or formulas relating percentage of possible sunshine or cloud cover to the percent of possible solar radiation which reaches the ground (see, for example, List, 1963, Tables 151, 152; Anderson, 1954, p. 74-77; Jensen and Haise, 1963, App. I). Bolsenga (1964) has provided tables of daily sums of cloudless sky short-wave radiation for various latitudes and air mass conditions which can be used as a basis for such estimates. For the purposes of the present paper, the amount of incident short-wave radiation  $Q_{RI}$  is estimated using the relation initially suggested by Angstrom (List, 1963):

$$Q_{RI} = Q_{CL} [0.35 + 0.061(10 - C)] \quad (14)$$

where  $Q_{CL}$  is the incoming short-wave radiation for a cloudless sky and  $C$  is the cloudiness expressed in tenths (i. e., with complete cloud cover,  $C = 10$ ).

A portion of the short-wave radiation which reaches the water surface is reflected, and this must be subtracted from the incident radiation to find the energy actually absorbed. Studies at Lake Hefner, Oklahoma (Anderson, 1954, p. 78-88) showed that water surface albedo ranged from 0.05 to 0.30, was a strong function of solar altitude, and depended to some extent on cloud conditions. Unfortunately, it is difficult to use Anderson's relations because the empirical constants must be changed to correspond with the type of cloud cover. Koberg (1964) has suggested a way around this problem by plotting curves for reflected short-wave radiation versus measured short-wave radiation for both clear and cloudy skies. In the present paper the amount of reflected short-wave radiation  $Q_{RR}$  was estimated by fitting a polynomial to average values between Koberg's clear and cloudy sky curves:

$$Q_{RR} = 0.108 Q_{RI} - 6.766 \times 10^{-5} Q_{RI}^2 \quad (15)$$

## THE EFFECTS OF THERMAL POLLUTION ON RIVER ICE CONDITIONS 9

Koberg estimates that this should be accurate to within 10%. The amount of energy absorbed by the water is then calculated by

$$Q_R = Q_{RI} - Q_{RR} \quad (16)$$

Under conditions where  $T_w > T_a$ , and windspeed is low, fog generally occurs. Such conditions are more likely the greater the difference ( $T_w - T_a$ ) becomes.  $Q_R$ , therefore, is commonly appreciably less than  $Q_{CL}$  and contributes only slightly to the total energy budget.

### $Q_B$

The net heat lost from the water surface as long-wave radiation is given by

$$-Q_B = Q_a - Q_{ar} - Q_{bs} \quad (17)$$

where  $Q_a$  is the incoming long-wave atmospheric radiation,  $Q_{ar}$  is the reflected long-wave atmospheric radiation, and  $Q_{bs}$  is the back radiation from the water surface. The Lake Hefner studies (Anderson, 1954, p. 96-98) have established that the emissivity of water is 0.970 and is independent of the temperature and concentration of dissolved solids. Thus

$$Q_{bs} = 0.970 \sigma T_w^4 \quad (18)$$

where  $\sigma$  is the Stefan-Boltzmann constant ( $1.171 \times 10^{-7} \text{ cal cm}^{-2} \text{ day}^{-1} \text{ } ^\circ\text{K}^{-4}$ ) and  $T_w$  is water temperature in  $^\circ\text{K}$ .

Where  $Q_a$  is not measured, it can be related empirically to air temperature, vapor pressure, and cloud conditions (Anderson, 1954, p. 88-98). For clear skies the relation initially suggested by Brunt is used with the empirical constants determined by Anderson:

$$Q_a = (0.68 + 0.036 \sqrt{e_a}) \sigma T_a^4 \quad (19)$$

where  $T_a$  is the air temperature in  $^\circ\text{K}$ ,  $e_a$  is the vapor pressure of the air in mb, and  $Q_a$  is expressed in  $\text{cal cm}^{-2} \text{ day}^{-1}$ . For cloudy skies  $Q_a$  was estimated by

$$Q_a = (a + b e_a) \sigma T_a^4 \quad (20)$$

where

$$a = 0.740 + 0.025 C \exp [(-1.92)(10^{-4})H] \quad (21)$$

$$b = (4.9)(10^{-3}) - (5.4)(10^{-4}) C \exp [(-1.97)(10^{-4})H] \quad (22)$$

and

$$500 \leq H < \infty.$$



## 10 THE EFFECTS OF THERMAL POLLUTION ON RIVER ICE CONDITIONS

Here  $C$  is the cloudiness expressed in tenths and  $H$  is the cloud height in meters. For cloud heights less than 500 m, the cloud height is taken as a constant at 500 m (Anderson, 1954). Because the emissivity of water is 0.970, its long-wave reflectivity is 0.03, so that  $Q_{ar} = 0.03 Q_a$ .

If weather observations are available, the value of  $e_a$  can be computed from

$$e_a = \frac{U e_{sa}}{100} \quad (23)$$

where  $U$  is the relative humidity and  $e_{sa}$  is the saturation vapor pressure of the air at temperature  $T_a$ . The value of  $e_{sa}$ , which is a function of temperature alone, can be calculated with the Goff-Gratch formula (List, 1963, p. 350).

### $Q_E + Q_H$

In studies of this type,  $Q_E + Q_H$  is generally estimated by using an empirical formula to calculate either  $Q_E$  or  $Q_H$ , and then using the Bowen ratio

$$R = \frac{Q_H}{Q_E} \quad (24)$$

to find the other. The value of  $R$  is calculated from

$$R = (6.1)(10^{-4}) P \left[ \frac{T_w - T_a}{e_{sw} - e_a} \right] \quad (25)$$

where  $P$  is atmospheric pressure (mb),  $e_{sw}$  is saturation vapor pressure at the temperature of the water (mb),  $e_a$  is vapor pressure of the air (mb), and other symbols are as previously defined.

Equation 25 is derived on the assumption that the eddy diffusivities of water vapor ( $\kappa_w$ ) and heat ( $\kappa_h$ ) in air are equal. Examination of previous literature reveals uncertainty about the conditions under which this assumption holds true. Pasquill (1949), Veihmeyer (1964) and Fritschen (1965) all reported satisfactory results using eq 25 when atmospheric conditions are neutral or stable. Anderson (1954, p. 117) also concluded as a result of the Lake Hefner studies that little error appears to be introduced in the estimates as the result of neglecting the effects of atmospheric stability. Brutsaert (1965, p. 13) cites one report in which  $\kappa = \kappa_h / \kappa_w$  was found equal to unity regardless of atmospheric stability, and another where the ratio was found equal to unity except under highly stable conditions, where it increased to 2 or 3. In unstable conditions ( $T_w \gg T_a$ ), Pasquill (1949) reported that  $\kappa$  could be as large as 3. A similar increase in the value of  $\kappa$  with an increase in ( $T_w - T_a$ ) was noted by Roll (1965, Fig. 81). However, Rimsha and Donchenko (1957), whose work is discussed more fully below, successfully used the Bowen ratio with  $\kappa = 1$  to calculate wintertime heat losses from calorimeters and open river reaches, with ( $T_w - T_a$ ) as great as 30°C. In view of these latter studies, it seems reasonable to use the

# THE EFFECTS OF THERMAL POLLUTION ON RIVER ICE CONDITIONS 11

Bowen ratio in calculating  $Q_E$  and  $Q_H$  under the conditions of interest here. Since  $\pi$  enters the theoretical derivation of  $R$  as a product term, the lack of information as to its true value can introduce considerable uncertainty in the estimates of  $Q_H$  or  $Q_E$ , and hence in the total heat loss rate. The possibilities of such error must be kept in mind in situations such as discussed herein.

Using the Bowen ratio, two approaches to calculating  $Q_E$  and  $Q_H$  were attempted. The first involves using a simple, well-accepted formula for calculating daily evaporation rates (Kohler, 1954).

$$E = [(0.525)(10^{-2}) + (1.007)(10^{-2}) v_a] (e_{sw} - e_a) \quad (26)$$

where  $E$  is evaporation rate ( $\text{cm day}^{-1}$ ),  $v_a$  is average wind velocity at the 4-m level ( $\text{m sec}^{-1}$ ),  $e_{sw}$  is saturation vapor pressure at the temperature of the water surface (mb), and  $e_a$  is the vapor pressure of the air at the 2-m level (mb). Multiplication of eq 26 by suitable constants allows the heat lost as a result of evaporation to be calculated by

$$Q_E = (3.135 + 6.011 v_a)(e_{sw} - e_a) \quad (27)$$

where  $Q_E$  is in  $\text{cal cm}^{-2} \text{ day}^{-1}$ ,  $v_a$  is in  $\text{m sec}^{-1}$ , and  $e_{sw}$  and  $e_a$  are in mb. Assuming that eq 27 is applicable within small limits of error when 2-m wind velocities are used, and that  $P = 1000$  mb, its multiplication by eq 25 yields\*

$$Q_H = (1.91 + 3.67 v_a)(T_w - T_a). \quad (28)$$

The second approach uses the equations developed by Rimsha and Donchenko (1957), who assumed that  $Q_H$  could be calculated by a relation of the form of eq 28, and determined the wind function parameters by least squares analyses of data from calorimeters exposed adjacent to natural ice-free reaches. The principal difference in the two approaches lies in the fact that the "constant" in the wind function was found by Rimsha and Donchenko to vary with  $(T_w - T_a)$ . Thus, after adjustment for application to natural reaches (based on actual comparison of heat losses from rivers and calorimeters), they wrote

$$Q_H = (k_n + 3.9 v_a)(T_w - T_a) \quad (29)$$

where  $v_a$  is 2-m windspeed in  $\text{m sec}^{-1}$  and  $k_n$  is empirically found to be

$$k_n = 8.0 + 0.35(T_w - T_a). \quad (30)$$

\* If changes are made in eq 26, 27 and 28 to correct for the substitution of average wind velocities determined at the 2-m level in equations based on 4-m wind velocities, these equations change to

$$E = [(0.525)(10^{-2}) + (1.229)(10^{-2}) v_a] (e_{sw} - e_a) \quad (26a)$$

$$Q_E = (3.135 + 7.33 v_a)(e_{sw} - e_a) \quad (27a)$$

and

$$Q_H = (1.91 + 4.47 v_a)(T_w - T_a). \quad (28a)$$

## 12 THE EFFECTS OF THERMAL POLLUTION ON RIVER ICE CONDITIONS

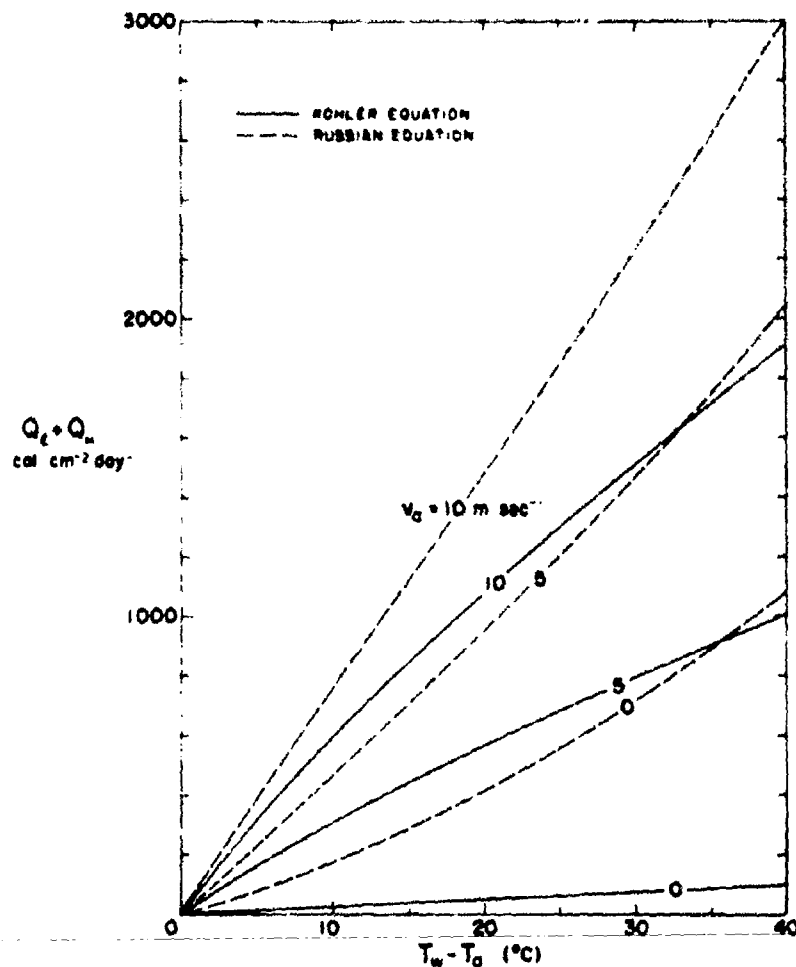


Figure 2.  $(Q_E + Q_H)$  vs  $(T_w - T_a)$  at  $v_a = 0, 5$ , and  $10 \text{ m sec}^{-1}$ ,  $U = 100\%$ . The different curves are calculated by the Russian winter equation and by the Kohler equation method.

The parameter  $k_n$  thus represents the portion of heat transfer due to free convection. The Bowen ratio, assuming  $P = 1000 \text{ mb}$ , was then applied by Rimsha and Donchenko (1957) to eq 29 to give

$$Q_E = (1.56 k_n + 6.08 v_a) (e_{sw} - e_a). \quad (31)$$

It is interesting to note that the wind coefficient in eq 31 is very close to that found by Kohler (1954, eq 27). Thus eq 27 is a special case of eq 31, which is, according to eq 30, appropriate for  $(T_w - T_a) < 1\text{C}$ .

Figure 2 compares  $Q_H + Q_E$  as calculated by eq 27 and 28 with that calculated by eq 29 and 31, and illustrates the large differences which arise from introduction of a term accounting for natural convection. The differences are, as expected, most pronounced at low windspeeds.

It is recognized that eq 26-31 are simplifications, and that the constants in each actually depend upon such factors as wind sheltering, fetch

## THE EFFECTS OF THERMAL POLLUTION ON RIVER ICE CONDITIONS 13

lengths, and the amplitude and frequency of wind variations at the sites for which they were determined. Some field tests are described below, but many more are needed before the general applicability of one or the other set of equations is demonstrated.

Figure 3 shows the relative importance of  $Q_B$ ,  $Q_E$  and  $Q_H$  under differing meteorological conditions using eq 29, 30 and 31.

### $Q_S$

The heat lost as a result of snow falling into the water  $Q_S$  ( $\text{cal cm}^{-2} \text{ day}^{-1}$ ) can be estimated by using the relation

$$Q_S = A[L + C_i(T_w - T_a)] \quad (32)$$

where  $A$  is the snow accumulation rate ( $\text{g cm}^{-2} \text{ day}^{-1}$ ),  $L$  is the latent heat of fusion of ice ( $\text{cal g}^{-1}$ ), and  $C_i$  is the heat capacity of ice. If possible,  $A$  should be taken from direct measurements of snowfall rate. In many cases these do not exist, but estimates of visibility will be available, and a satisfactory estimate may be possible using a relationship such as that suggested by Mellor (1966):

$$A = 7.85V^{-2.375} \quad (33)$$

where  $V$  is visibility in km. Equation 33 was found to hold reasonably well over the range  $1 \leq V \leq 10$  under calm conditions. A feel for the relative importance of  $Q_S$  in contributing to the value of  $Q^*$  can be obtained by comparing Figure 4 with Figure 3.

### $Q_G$

An exact value for this term requires a knowledge of the geothermal gradient in the area of concern and the thermal diffusivity of the stream bed material. Lacking this, values measured under similar conditions may be found in the literature. The only reports found were by Ince and Ashe (1964), who indicated that, in December,  $Q_G \approx 5 \text{ cal cm}^{-2} \text{ day}^{-1}$  for some Swedish rivers and by Rimsha and Donchenko (1957) who gave  $Q_G \approx 10$  to  $20 \text{ cal cm}^{-2} \text{ day}^{-1}$  for some Russian rivers. Both of these values can undoubtedly be considered high inasmuch as a representative heat flow value for the Canadian Shield is roughly  $0.1 \text{ cal cm}^{-2} \text{ day}^{-1}$  (Lee and Uyeda, 1965). Thus in most cases it is safe to consider  $Q_G$  negligible.

### $Q_{GW}$

Fairly detailed knowledge of the river is required to evaluate this term. Stream discharge hydrographs can be analyzed to provide a rough estimate of gains or losses attributable to ground water. If it proves necessary to consider ground water, the mean annual air temperature at the site should provide a reasonable estimate of the temperature of the shallow ground water. Since, if ground water flow is significant, assumptions of constant discharge are violated,  $Q_{GW}$  is considered negligible.

# 14 THE EFFECTS OF THERMAL POLLUTION ON RIVER ICE CONDITIONS

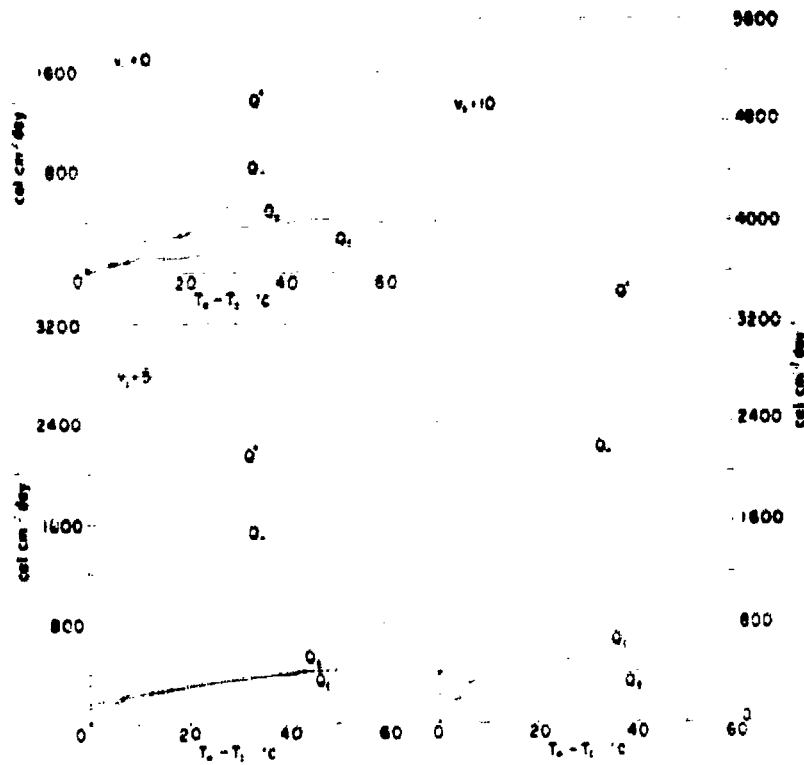


Figure 3.  $Q_B$ ,  $Q_E$ ,  $Q_H$ , and  $Q^*$  vs  $(T_w - T_a)$  at  $v_a = 0, 5$ , and  $10 \text{ m sec}^{-1}$ ;  $U = 50\%$ ; clear-sky conditions.  $Q_E$  and  $Q_H$  calculated by the Russian winter equation and Bowen ratio.

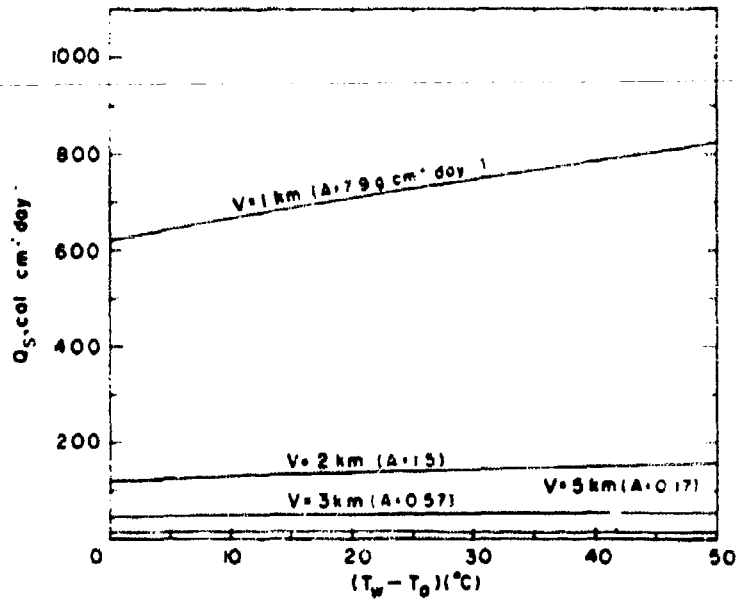


Figure 4.  $Q_S$  vs  $(T_w - T_a)$  at  $v_a = 0$ .

$$\underline{Q_F}$$

The heat added due to fluid friction, can be calculated using a relation given by Ince and Ashe (1964, p. 8):

$$Q_F = 8.64 \times 10^4 \frac{D\gamma S}{Jw} \quad (34)$$

where  $D$  is river discharge ( $\text{cm}^3 \text{sec}^{-1}$ ),  $\gamma$  is weight density of water ( $\text{dy cm}^{-3}$ ),  $S$  is the slope of the water surface,  $w$  is the width of the river ( $\text{cm}$ ) and  $J$  is the thermal equivalent ( $4.187 \times 10^7 \text{ dy cm cal}^{-1}$ ). Under most conditions,  $Q_F$  is insignificant.

### CALCULATION OF X

The distance downstream from the site of thermal pollution to the cross-section at which the water temperature has decreased to a given value is found by substituting the chosen expressions for the heat budget components (eq 14-34) in eq 11 and integrating between suitable limits. It is clear that if the meteorological parameters  $T_a$ ,  $v_a$ ,  $e_a$ ,  $Q_{CL}$ ,  $V$ ,  $C$ ,  $H$  and  $P$  can be considered as known constants over the reach of river of interest,  $Q^*$  is a function of water temperature only. Unfortunately, the function

$$\int dT_w / Q^*$$

cannot be integrated in closed form and numerical integration must be used.

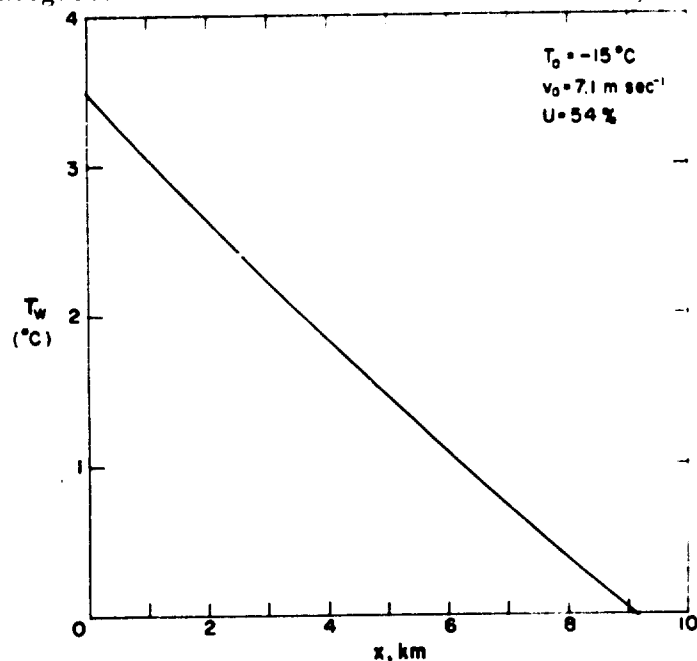


Figure 5. Typical downstream temperature profile below a thermal pollution source.

A computer program was prepared to provide a means of evaluating eq 11 by using Simpson's rule. The program was prepared so that calculations could be performed for any combination of values of the hydrometric and meteorological parameters. The water temperature at the site of thermal pollution ( $x = 0$ ) was calculated using eq 12. Simpson's rule was

## 16 THE EFFECTS OF THERMAL POLLUTION ON RIVER ICE CONDITIONS

then used to compute the value of the integral and the corresponding value of  $x$  in 0.1 K steps downstream until some cutoff temperature was reached. In the problems considered in this paper the cutoff temperature was 273.2 K, the ice point.

Figure 5 shows the form of a typical temperature vs distance curve. The program may, of course, also be used to calculate temperature profiles downstream from thermal pollution sites under meteorological conditions such that no ice will form, as long as the river is cooling at the surface.

### "EXPERIMENTAL" CHECKS

It is difficult to state the accuracy with which the approach outlined in this paper will permit the calculation of the position of the ice front. Besides the difficulty in accurately predicting  $Q^*$ , it is obvious that an ice sheet will not form exactly at the point where the water temperature reaches 0°C. Under conditions of intense turbulent mixing, frazil ice may form initially. Where vertical mixing is weak, the theoretical position of the ice sheet will in general also lie upstream from the actual ice front.

It should be emphasized that this paper calculates only the steady-state ice-free reach. The calculations should, therefore, be most representative when the weather conditions and  $Q_p$  have been relatively constant. When either the air temperature or  $Q_p$  are decreasing with time the calculations should also be representative, because the position of the ice front should rapidly adjust to the new steady-state value. However, the length of the calculated ice-free reach may be in considerable error when an appreciable cold period is followed by a warming trend. During the cold period, ice will accumulate on the river below the position of the ice front. When the air temperature rises, some of this ice must melt as the length of the ice-free reach increases to the new steady-state value. The time required for this ice to melt will cause a time lag in achieving the new steady-state position of the ice front.

Because of these difficulties and because many of the assumptions involved in writing eq 11 are not strictly true in a real situation, it is highly desirable to be able to compare field measurements of ice-free reaches below existing thermal pollution sites with computed values. Unfortunately, careful field observations of this phenomenon are extremely limited. However, it was possible to obtain unpublished data collected by the Corps of Engineers, St. Paul District,\* regarding the length of ice-free reaches below the Northern State Power Company's Riverside and Highbridge steam-power generating plants. These plants are located on the Mississippi River in the vicinity of Minneapolis-St. Paul and discharge thermal pollution directly into the river. As a test area this general reach of the Mississippi has several disadvantages: several other small plants discharge thermal pollution and small tow boats frequently pass up and down the river when ice conditions are not too severe. Both these factors undoubtedly disturb the natural development of an ice sheet. The extent to which they enlarge the ice-free reaches below the Riverside and Highbridge plants is unknown. Further, at a point 5 km below the Riverside plant there is a lock and dam.

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\* M. Nelson, personal communication.

# THE EFFECTS OF THERMAL POLLUTION ON RIVER ICE CONDITIONS 17

Table I. Observed and calculated values for the areas and lengths of the ice-free reaches below thermal pollution sites along the Mississippi River, Minneapolis-St. Paul, Minnesota.

Date	Riverside				Highbridge			
	Observed area (km <sup>2</sup> )	Observed length (km)	Calculated length		Observed area (km <sup>2</sup> )	Observed length (km)	Calculated length	
			Russian winter eq (km)	Kohler eq (km)			Russian winter eq (km)	Kohler eq (km)
30 Jan 66	2.109	12.0	17.9	96.6	4.597	40.0	42.6	62.5
29 Jan 66	.777	4.3	4.3	7.0	.882	7.6	11.8	18.6
4 Feb 66	.890	4.9	5.8	10.0	1.036	9.0	15.1	26.6
11 Jan 66	---	---	---	---	2.428	21.2	15.7	21.8
14 Jan 66	2.023	11.3	16.0	28.9	4.233	36.8	41.3	73.2
34 Jan 66	1.311	7.3	3.2	5.8	1.546	13.4	12.1	21.2
28 Jan 66	1.237	7.2	3.6	5.3	1.076	9.4	10.4	13.8
23 Feb 66	2.161	12.0	17.1	60.8	5.063	44.0	39.5	124.6

complex, and 6.4 km below the Highbridge plant a significant amount of thermal pollution is introduced by the Minneapolis-St. Paul Sanitary District plant. Therefore neither of these locations represents an ideal test site (i.e., a location where an isolated thermal pollution source introduces heat into a river that possesses relatively constant hydrometric parameters).

The total area of open water below these two plants was determined from aerial photographs on several different dates during the winters of 1965 and 1966. Because the open water - ice boundary was usually quite irregular, the area of open water was converted to an effective length by dividing it by a representative width of the river. Below the Riverside plant the full width of the river was used. However, below the Highbridge plant an effective width of one-half the actual width was utilized because field observations indicated that thermal mixing was confined to the left half of the river. The meteorological data used are from the U.S. Weather Bureau Station at the Minneapolis-St. Paul Airport (average daily values), and the daily average thermal pollution values ( $Q_p$ , cal sec<sup>-1</sup>) for the Riverside, Highbridge and Minneapolis-St. Paul Sanitary District plants were provided by the St. Paul District, U.S. Army Corps of Engineers. These data are tabulated in Appendix C.

The question immediately arises as to the time period over which the meteorological conditions should be averaged or, stated another way, how quickly the length of the open water reach responds to changes in meteorological conditions. While no study was undertaken to resolve this matter, Bilello's (1967) study of Seneca Lake, Michigan, suggests that near-surface water temperature trends follow air temperature trends with a lag of about 1 day. The work of Ince and Ashe (1964) on the St. Lawrence River suggests a similar response time. In most cases considered in the present paper, the length of time required for the water to traverse the ice-free reach is less than or only slightly greater than 1 day. Therefore, the average meteorological data from the date prior to the date of ice observation were used in calculating the length of the ice-free reach.

Table I presents the observed ice-free areas below the thermal pollution sites and "observed" ice-free lengths calculated from these areas. Ice-free lengths calculated using the Kohler equation method and the Russian winter



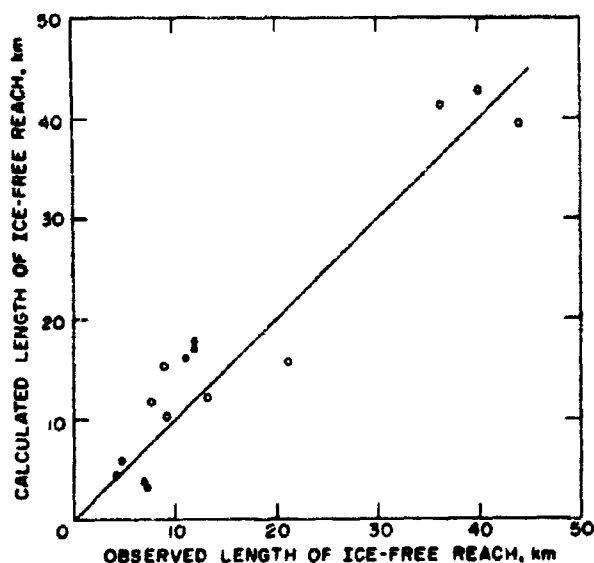


Figure 6. Observed vs calculated ice-free reaches. Measurements below the Highbridge Plant are shown as open circles and measurements below the Riverside Plant as dots.

equation are also listed. In the calculations regarding the Highbridge plant, the calculated open reach is longer than the distance (6.4 km) to the next large thermal pollution source, the Minneapolis-St. Paul Sanitary District plant. Therefore, to calculate the total length of the open reach, the calculated water temperature 6.4 km below the Highbridge plant was taken as the natural water temperature  $T_{nat}$  when the thermal pollution associated with the Minneapolis-St. Paul Sanitary District plant is introduced. The final length of the open reach produced by both plants is then simply the length of the calculated reach below the Minneapolis-St. Paul Sanitary District plant plus 6.4 km.

As would be expected from examining Figure 2, the lengths calculated by the Russian winter equation are always shorter than those calculated by using the Kohler equation method. They are also clearly in better agreement with the observed lengths. Figure 6 presents a plot of the observed lengths of the ice-free reaches against the lengths calculated using the Russian winter equation. The agreement is very encouraging. Although there is a slight tendency for the calculated lengths to be longer than observed, no consistent pattern is apparent.

## APPLICATIONS TO THE ST. LAWRENCE SEAWAY

The ship traffic on the St. Lawrence Seaway is completely stopped by ice for  $3\frac{1}{2}$  to 5 months each winter. Particular difficulty is encountered in the vicinity of the lock areas and ship canals. If these areas could be kept ice-free it would be possible to move shipping into the Great Lakes during most of the winter. This would be a great convenience and would save transportation costs of several million dollars per year. One conceivable way to accomplish this is to use the excess heat produced by nuclear power reactors to remove ice at critical locations. The production of electricity by nuclear power is now becoming commercially competitive with conventional forms of power production. In addition, the exact positioning of power reactors is not important as long as there is a major river nearby that can be used as a source of coolant and the reactor can be coupled with the general power grid system currently in use in eastern Canada and the United States. If it can be shown that the thermal pollution from a reactor is sufficient to keep a significant length of a shipping canal ice-free during the winter, a strong case can be made for positioning future reactors along the St. Lawrence River.

Two types of nuclear reactors are considered here: those presently available with a thermal pollution output of  $2.9 \times 10^8 \text{ cal sec}^{-1}$  ( $\sim 1.2 \times 10^9$  watts) and those in the design stage with a thermal output of  $8.4 \times 10^8 \text{ cal sec}^{-1}$  ( $\sim 3.5 \times 10^9$  watts). As an estimate of extremely severe winter conditions for the general seaway area, a value of  $T_a = -17^\circ\text{C}$  was used. This is based on the mean monthly minimum temperatures from Ottawa (January,  $-16.7^\circ\text{C}$ ; February,  $-15.6^\circ\text{C}$ ; 31 years of record) and Quebec (January,  $-16.1^\circ\text{C}$ ; February,  $-15.6^\circ\text{C}$ ; 32 years of record). A windspeed of  $5 \text{ m sec}^{-1}$  was assumed as representative, based on January and February data from Cornwall, Ontario. This windspeed is undoubtedly high because in the general St. Lawrence area low temperatures are usually associated with relatively calm conditions and pronounced radiational cooling. A representative average temperature for the seaway area during January and February is  $-11^\circ\text{C}$ .

As an example the ship canal around the Montreal area is considered: the South Shore Canal with the St. Lambert and Cote St. Catherine Locks. In the fall and early winter the water temperatures in this area are 1 to 2 degrees lower than upstream in the vicinity of Cornwall; therefore, the South Shore Canal is the first location on the Seaway that encounters significant ice problems. The average width of the canal is 150 m, the depth is 9 m and the water flow velocity  $0.5 \text{ m sec}^{-1}$ . The length of the South Shore Canal from Montreal Harbor to Lake St. Louis is roughly 32 km. As can be seen from Figure 7 the lengths of the Canal that can be kept ice-free with  $T_a = -17^\circ\text{C}$ ,  $v_a = 5 \text{ m sec}^{-1}$  and  $C = 10$  are 17 (Russian winter) and 25 km (Kohler's equation) using a conventional reactor. With a larger reactor the calculated lengths are 50 and 75 km respectively. If similar estimates are made by using the Russian winter equation and the average winter temperature of  $-11^\circ\text{C}$ , values of 27 and 75 km are obtained for the conventional and design reactors respectively. It is obvious that even a conventional reactor would considerably lengthen the shipping season, while the larger reactor would effectively keep the canal open all winter.

These figures indicate the definite feasibility of using nuclear reactor power plant cooling effluent to keep selected portions of the St. Lawrence Seaway ice-free for much of the winter. The prevention of ice formation in the vicinity of locks would be especially beneficial.

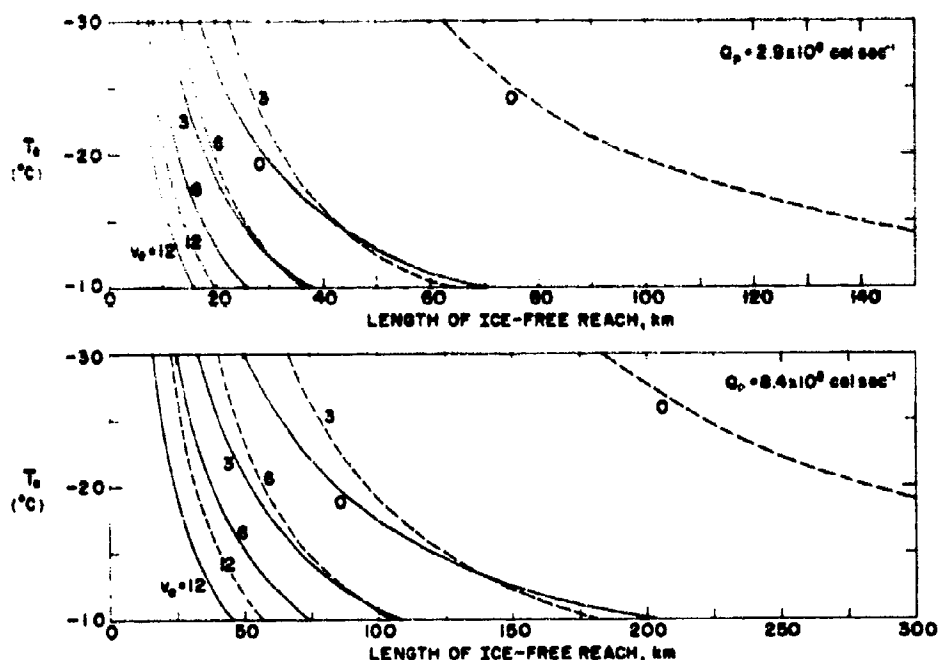


Figure 7. Length of ice-free reach as a function of air temperature ( $^{\circ}\text{C}$ ) and wind speed ( $\text{m sec}^{-1}$ ): South Shore Canal, St. Lawrence Seaway. Solid curves are based on the Russian winter equation and the dashed curves on the Kohler equation method.

### CONCLUSIONS

The general method for calculating the downstream temperature profile of a river and the length of reach which can be kept ice-free by artificial heat addition was stated, but not developed, by Pruden et al. (1954). They and previous workers did not develop a universally applicable expression for the heat loss rate  $Q^*$  which could be used with daily averages of the meteorologic variables. The present paper derives the basic temperature-distance relation from fundamental considerations and, by use of a computer program, allows the use of several of the many previously developed empirical and semiempirical formulas for the heat budget terms comprising  $Q^*$ . The computer program thus makes possible more universally applicable estimates of temperature profiles and lengths of ice-free reaches than were heretofore practicable. It has at least three important additional advantages: 1) temperature profiles and lengths of ice-free reaches can be quickly calculated for any combination of heats of pollution and meteorologic and hydrometric variables; 2) downstream changes in meteorologic and hydrometric variables can be accounted for, if desirable, by dividing the river into any number of sub-reaches and treating the water temperature at the outlet of one sub-reach as the temperature at the beginning of the next sub-reach downstream; and 3) the computer program can be easily modified to permit the use of different expressions for the heat budget items or the addition of other heat budget terms which may be significant in a particular case.

The important problems that must be overcome to improve the "accuracy" of the present method of calculation are the limited understanding of the evaporative and convective heat loss processes when an unstable vertical density distribution exists over open water, and the lack of a general method for evaluating the vertical and especially the horizontal mixing of heat in a

## THE EFFECTS OF THERMAL POLLUTION ON RIVER ICE CONDITIONS 21

river. It is, however, encouraging that observations on the Mississippi River at Minneapolis-St. Paul show reasonable agreement with computed lengths of ice-free reaches. Better tests of the method are highly desirable.

Calculations based on representative values of the meteorologic and hydro-metric variables for the South Shore Canal portion of the St. Lawrence Seaway suggest that significant reaches of that waterway could be kept ice-free by the proper placement of nuclear reactors. Significant lengthening of the shipping season with concomitant economic advantages would thus be feasible.

Although the current computer program is not designed to handle situations where the air temperature is warmer than the water temperature, it can easily be modified to include such situations. If this were done, the program would be useful in treating such problems as the cumulative effect of the positioning of a number of nuclear reactors along the Columbia River on the temperature of the river (Geraghty, 1967).

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## 22 THE EFFECTS OF THERMAL POLLUTION ON RIVER ICE CONDITIONS

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### Symbology

The postscript ---IN denotes the initial value of the variable ---, the postscript ---DEL indicates the incremental value of ---, and the postscript ---F indicates the final value. The variable ---INC denotes the initial value of --- in degrees centigrade, etc. The units to be used for the variables are as listed in the List of Symbols. Because the ---DEL values are always positive, ---IN should be a smaller value than ---F (e.g., VAIN = 0., VADEL = 5.0, VAF = 20.0., or TAINC = -50.0, TADEL = 10.0, TAF = -10.0, which correspond to TAIN = 223.2, TADEL = 10.0, TAF = 263.2). INT is the number of intervals used in the Simpson's rule calculation; a value of INT = 10 was found satisfactory.

### Sense Switch Instructions

#### Sense Switch 1

OFF:  $POLL = POLL + POLLDEL$  (POLLDEL is used and there is a constant additive increment).

ON:  $POLL = 10. * POLL$  (the program increments POLL by powers of 10). In this case any fixed point number is put in the POLLDEL site on the data tape.

#### Sense Switch 2

OFF: Not snowing (this option is given because decreased visibility does not necessarily indicate snow).

ON: Snowing.

#### Sense Switch 3

OFF: The Russian equations (eq 29-31) are used to calculate QE and QH.

ON: The Kohler equations (eq 27-28) are used to calculate QE and QH.

### Data Input

The data are introduced in the order and format specified in the program.

## APPENDIX A

```

C      WERNER-BINGHAM 2 WATER TEMPERATURE CALCULATOR
C      1 JUNE 1967 FORTRAN 2
C      DIMENSION A11(6), S1PP2(100)
C      COMMON CCL, PRES, PUG, TUNAT, TUFIAL, CLDC, CLDM, U, V, VA,
C      TAC, TAI, S1PP2, J
C      READ MES, AND MYPE, DATA
C      READ PAPER TAPE 1, INT, OCL, PRES, PUGIN, PUGDEL, PUGF,
C      TUNATC, MIN, MCEL, MP, LIN, VEEL, VP, VVIN, VVDEL, VVP
C      READ PAPER TAPE 2, CLEGIN, CLECEEL, CLDCP, CLDMIN, CLDMDEL,
C      TLELPP, LIA, LIEL, LP, VIN, VDEL, VP
C      READ PAPER TAPE 3, VAIN, VAEEL, VAP, TAINC, TADEL, TAPC
C      READ PAPER TAPE 4, PCLLIN, POLLEEL, POLLF
C      FORMAT(10, 15F7.0)
C      FORMAT(112F8.0)
C      FORMAT(10F7.0)
C      FLAP(13E20.0)
C      INITIALIZE VARIABLES
C      PUG = PUGIN
C      M = MIN
C      U = LIN
C      V = VIN
C      VA = VAIN
C      CLEC = CLEGIN
C      CLEP = CLDMIN
C      U = LIN
C      V = VIN
C      VA = VAIN
C      POLL = PCLLIN
C      TAP = TAPC + 273.2
C      TAIN = TAINC + 273.2
C      TA = TAIN
C      TUNAT = TUNATC + 273.2
C      CALCULATE INITIAL WATER TEMP.
C      TUPCLL = (1.2-DE/POLL)/(IN+U+V)
C      TUPCLL = 10.0*TUPCLL + 0.5
C      I = TUPCLL
C      TUPCLL = 1
C      TUPCLL = TUPCLL/10.
C      TWIN = TUNAT + TUPCLL
C      TWINC = TWIN + 273.2
C      PRINT VARIABLES USED IN CALCULATIONS
C      TAC = TA - 273.2
C      PRINT 5
C      FORMAT(11H 3X 5P POLL 5X 5M TUNAT 4X 7M TUPCLL 3X 5M TWIN)
C      PRINT 6, POLL, TUNATC, TUPCLL, TWINC
C      FORMAT(1E12.5, 3F10.2/)
C      PRINT 7
C      FORMAT(1H 4X 2P U 5X 2P V 7X 3M VM 5X 2M N 5X 2M P)
C      PRINT 8, U, V, VM, PUG, PRES
C      FORMAT(15F10.2/)
C      PRINT 9
C      FORMAT(11H 3X 4P CCL 5X 5M CLEC 5X 5M CLDM 7X 2M U 5X
C      1 2P V 7X 3M VA 7P 3M TAI)
C      PRINT 10, CCL, CLEC, CLDM, U, V, VA, TAC
C      FORMAT(17F10.2)
C      DECIDE IF SACKING
C      IF (SENSE SWITCH 2100.02
C      PRINT 61
C      FORMAT(11H 4M SACKING)
C      DECIDE IF USING NEHLER EQUATION
C      IF (SENSE SWITCH 3103.05
C      PRINT 64
C      FORMAT(11H 27P CE + 5H BY NEHLER EQUATION.////)
C      CE TC 71
C      USING RUSSIAN EQUATION
C      PRINT 67
C      FORMAT(11H 20P CE + 5H BY RUSSIAN EQUATION.////)
C      ROLAC OFF WATER TEMPERATURES, SET LIMITS FOR SIMPSONS
C      RULE CALCULATIONS
C      TUNALU = TUNAT - 1.
C      TUNAT = TUNAT/10.
C      TUNAT = TUNAT
C      TUNATC = TUNAT - 273.2
C      TUNALC = TUNAT - 273.2
C      TUNAL = TUNALC - 273.2
C      SET SUPR = C. PRINT COLUMN HEADINGS

```

# APPENDIX A

25

```

      SUPA = 0.
      PRINT 11
11  FORMAT (1P, 1P, TUNIT, 3P, 7P, TUPIAL, 13P, 2P, 2, 12P, 5P, DELTA)
C   CALL SIMPSONS ALLEA, CALCULATE AND PRINT DISTANCE AND
C   WATER TEMPERATURE
      GO TO J = 1, A = 1
      CALL SIMP2LA
      RIJ1 = -4.04E00+0000+SIMPP21J1
      SUMT = SUMT + RIJ1
      PRINT 13, TUNITC, TUPIALC, SUMT, RIJ1
13  FORMAT (12P10.2, 2E20.0)
      TUNIT = TUNIT + 0.1
      TUPIAL = TUPIAL + 0.1
      TUNITC = TUNIT + 273.2
      TUPIALC = TUPIAL + 273.2
12  INCREMENT VARIABLES
C   IF 17A - 14P1 18, 19, 19
      IF 17A - 14P1 18, 19, 19
18  TA = TA + TACEL
      GO TO 20
19  TA = TAIN
      IF 17A - 14P1 21, 22, 22
21  VA = VA + VACEL
      GO TO 20
22  TA = TAIN
      VA = VAIN
      IF 17A - 14P1 23, 24, 24
23  V = V + VPEL
      GO TO 20
24  TA = TAIN
      VA = VAIN
      V = VIN
      IF 17A - 14P1 25, 26, 26
25  L = L + LCEL
      GO TO 20
26  TA = TAIN
      VA = VAIN
      V = VIN
      L = LIN
      IF 17A - 14P1 27, 28, 28
27  CLEN = CLEN + CLECEL
      CLEN = CLEN + CLENDEL
      GO TO 20
28  TA = TAIN
      VA = VAIN
      V = VIN
      L = LIN
      CLEN = CLEN + CLECEL
      IF 17A - 14P1 29, 30, 30
29  CLEC = CLEC + CLECEL
      GO TO 20
30  TA = TAIN
      VA = VAIN
      V = VIN
      L = LIN
      CLEN = CLEN + CLECEL
      CLEC = CLEC + CLECEL
      IF 17A - 14P1 31, 32, 32
31  VM = VM + VDEL
      GO TO 20
32  TA = TAIN
      VA = VAIN
      V = VIN
      L = LIN
      CLEN = CLEN + CLECEL
      CLEC = CLEC + CLECEL
      VM = VM + VDEL
      IF 17A - 14P1 33, 34, 34
33  U = U + UDEL
      GO TO 20
34  TA = TAIN
      VA = VAIN
      V = VIN
      L = LIN
      CLEN = CLEN + CLECEL
      CLEC = CLEC + CLECEL
      VM = VM + VDEL
      U = U + UDEL

```



```

22  IF (N = 1) 37, 38, 39
    M = P * PELL
    GO TO 50
36  TA = TAIN
    VA = VAIN
    V = VIN
    U = UIN
    CLEM = CLEMIA
    CLEC = CLECIA
    VC = VAIN
    W = WIN
    M = MIN
    IF (FUG = FUGI) 37, 38, 39
27  FLC = FLC + FUGI
    GO TO 20
38  TA = TAIN
    VA = VAIN
    V = VIN
    U = UIN
    CLEM = CLEMIA
    CLEC = CLECIA
    VC = VAIN
    W = WIN
    M = MIN
    FLC = FUGI
    IF (SENSE SWITCH) 1139, 40
40  IF (PCLL = PCLLF) 41, 43, 43
41  PCLL = PCLL + PCLLEL
    GO TO 50
39  IF (PCLL = PCLLF) 42, 43, 43
42  PCLL = 10. * PCLL
    GO TO 50
    STOP
    END
0

```

## SUBROUTINE SIMP20A

```

SUBROUTINE SIMP20A
C  SIMPSONS RULE INTEGRATION WITHIN EACH 0.1 C INCREMENT
  DIMENSION Y(100), SIMP2(100)
  COMMON CCL, PRES, FUG, TWINT, TUFINL, CLDC, CLDM, U, V, VA,
  ITA, IAT, SIMP2, J
C  CALCULATE FIRST AND LAST COORDINATE VALUES
  Y(1) = GINVICL, PRES, FUG, TWINT, CLDC, CLDM, U, V, VA, TA
  Y(IAT + 1) = GINVICL, PRES, FUG, TUFINL, CLDC, CLDM, U, V,
  (VA, TA)
C  CALCULATE DELTA
  ZINT = INT
  DELTA = (TWINT - TUFINL)/ZINT
C  CALCULATE EVEN-NUMBERED ORDINATES
  SUPEVEN = 0
  DO 100 I = 2, IAT, 2
    Y(I) = GINVICL, PRES, FUG, TWINT - DELTA*(I - 1), CLDC, CLDM,
    U, V, VA, TA
100  SUPEVEN = SUPEVEN + Y(I)
C  CALCULATE ODD-NUMBERED ORDINATES
  SUPCED = 0
  DO 101 I = 3, IAT - 1, 2
    Y(I) = GINVICL, PRES, FUG, TWINT - DELTA*(I - 1), CLDC, CLDM,
    U, V, VA, TA
101  SUPCED = SUPCED + Y(I)
C  CALCULATE VALUE OF INTEGRAL
  SIMP2(I) = (DELTA/3.)*(Y(1) + Y(IAT + 1) + 2.*SUMODD + 4.*
  (SUPEVEN)
  RETURN
  END
0

```

# APPENDIX A

27

```

C      FUNCTION KIN10CL, PMS, PGG, T0, PLPA, CLC, L, V, VA, TAI
C      CALCULATE 1/64
      J=0
      J=J+1
C      CALCULATE G1 (EGS, 14 = 14)
      W1 = 0.0101035 + 0.001110 = CLC11
      DRA = 10.10A-G1 = 0.700-0.0101035=0.689896
      W1 = W1 + LWR
C      CALCULATE G1 (EGS, 17 = 22)
      W1 = 0.0101035 + 0.001110 = CLC11
      EA = (L+VA*TAI)/100.0
      IF (CLC11) 200, 200, 201
200  G1 = 10.0A + 0.020+5GHTF (FAI)+5GHTF (TAI)
      GO TO 202
201  A = 0.74 + 0.020+5GHTF (FAI)+5GHTF (TAI)
      W1 = 0.0101035 + 0.001110 = CLC11
      G1 = 10.0A + 0.020+5GHTF (FAI)+5GHTF (TAI)
202  G1 = 0.0101035
      G1 = G1 + CA + G1A + G1B
C      CALCULATE G1 AND G1 USING ROMAN EG, AND ROMAN RATIO
C      (EGS, 27 = 28)
      IF (EGS) 301, 301, 203, 204
203  G1 = 10.135 + 0.010+VAI (VAPITAI) = EA
      G1 = 10.135 + 0.010+VAI (VAPITAI) = EA
      GO TO 208
204  CALCULATE G1 AND G1 USING ROMAN EG, AND ROMAN RATIO
C      (EGS, 29 = 31)
      W1 = 0.0101035 + 0.001110 = CLC11
      G1 = 10.135 + 0.010+VAI (VAPITAI) = EA
      G1 = 10.135 + 0.010+VAI (VAPITAI) = EA
C      CALCULATE G1 IF INCLING (EGS, 32 = 33)
201  IF (EGS) 301, 301, 203, 210
204  APTOTAS=1.700+0.335
      G1=AC=10.135+0.335
      GO TO 211
210  G1=0
C      CALCULATE G1 AND 1/64
211  G1=AC=10.135+0.335
      G1=1.700+0.335
      RETURN
      STOP
      END
      G

```

```

C      FUNCTION VAP (Z)
C      GOFF-GRATCH FCAPLLA FOR SATURATED VAPOR PRESSURE
      A = -7.90290+(373.16/Z) - 1.1
      B = 9.82800+PLC10F (373.16/Z)
      C = -1.3010E - 07*(10.00+11.344+(1.-2/373.16)) - 1.1
      E = 0.1320E - 03*(10.00+3.49149+(373.16/Z) - 1.1) - 1.1
      E = FLOG10F (1013.240)
      ALCEU = A + B + C + E + E
      VAP = 10.00+ALCEU
      RETURN
      END
      G

```

**APPENDIX B: COMPUTER PROGRAM FOR CALCULATING HEAT      29**  
**BUDGET COMPONENTS AND TOTAL HEAT LOSS**  
**RATES FROM A COOLING RIVER**

**Symbology**

Initial, incremental, and final values of variables are designated as described in Appendix A.

**Sense Switch Instructions**

Sense switches are used as in Appendix A, with the exception that Sense Switch 1 is not used.

**Data Layout**

The data are introduced in the order and format specified in the program. The FUNCTION VAP(Z) subroutine used in this program is identical to that used in Appendix A.

```

C      VEEPS=BINGMAN & GETAW GENERATOR
C      CALCULATES SH, SA, SC, CH, CS, AND CO
C      11 DEC 65 PORTLAND
C      COMPAR 10.1A
C      SET CONSTANTS
C      SHC = 1.171E-07
C      CI = 0.5
C      READ MET. DATA
27  PRINT PAPER TAPE 1, CCL,PRES,FUGIN,FLGDEL,FLGFC,
      ICLECIN,CLCEFL,CLCEH
      PRINT PAPER TAPE 2, CLEHIN,CLHDEL,CLHFL,CLHDFL,
      UFL,VIN,VDEL,VFL,VAIN,VDEL,VAF
      READ PAPER TAPE 3, TUINC,TUGFL,TUFC,TAINC,TADEL,
      TAF
1  FORMAT(9F9.0)
2  FORMAT(12F9.0)
3  FORMAT(9F9.0)
C      INITIALIZE MET. VARIABLES
      VARVAIA
      FUG=FUGIN
      CLEC=CLCEIN
      CLH=CLHIN
      UFLIA
      VINIA
      TUIN=TUINC+273.2
      TAIN=TAINC+273.2
      TAF=TAF+273.2
      TUFC=TUFC+273.2
      TU=TUIN
      TA=TAIN
13  TUC=TU+273.2
      TAC=TA+273.2
C      PRINT VARIABLES USED IN CALCULATIONS
      PRINT 4
4  FORMAT(10X 4P CCL 5X 5P PRES 6X 2P 4 6X 5P
      1 CLEC 5X 5P CLEH)
      PRINT 5, CCL,PRES,FUG,CLCE,CLH
5  FORMAT(1P F15.2+2F9.0+F10.1+F10.0/)
      PRINT 60
60  FORMAT (1P 11X 2P V 9X 2P U FL 3P VA 6X 3P TA 6X
      14P TUC)
      PRINT 70, V, U, VA, TU, TUC
70  FORMAT (1P F10.1, F10.1, F9.1, F10.2, F11.2/)
C      CALCULATES CR (EGS. 14 - 16)
      CH = CCL+10.35 + 0.001*(10. - CLEC)
      CRH=(0.109+CR1-0.766E-05+CR1**2.)
      CR=CR1-LWR
C      CALCULATES GB (EGS. 17 - 22)
      QAS=0.97+SBC+TA**4.
      EAPIL=VAP(TA)/100.
      IF(CLEC16.0.7
6  QA=(0.68+0.036*SQRT(EA1))+SBC+TA**4.
      GO TO 8
7  AHC.74+0.025*CLCE*EXPF(-1.92F-C4+CLCE)
      BH.4.9E-03-5.4E-04*CLCE*EXPF(-1.97E-C4+CLCE)
      GA=(A+B+EA)*SBC+TA**4.
8  QA=0.03+QA
      DB = -QA + CAR + CBS
C      CALCULATES GE AND GH USING KOHLER EG. AND BOWENS
      RATIO (EGS. 27 - 28)
C      IF (SENSE SWITCH 3) 203, 204
203 PRINT 202
202  FORMAT (1P 27P GE + GH BY KOHLER EQUATION,/)
      CF = (3.175 + C.C11+VA)*(VAP(TU) - EA)
      GH = (11.91 + 5.67+VA)*(TU - TA)
      GO TO 208
C      CALCULATES GE AND GH USING RUSSIAN EG. AND BOWENS
      RATIO (EGS. 24 - 31)
C      204 PRINT 205
205  FORMAT (1P 29P GE + GH BY RUSSIAN EQUATION,/)
      XRAH=.35*(TU-TA)
      GE = (1.5F+XRAH + C.C8+VA)*(VAP(TU) - EA)
      GH = (XRAH + 3.6+VA)*(TU - TA)
C      CALCULATES GS IF SNOUGH IFCS. 32 - 33)
208  IF (SENSE SWITCH 2) 30, 31
30  ACC = 7.89+11.7V**2.375)

```

## 31

[illegible]

**APPENDIX C: METEOROLOGICAL DATA, U.S. WEATHER BUREAU, 33  
MINNEAPOLIS-ST. PAUL AIRPORT (AVERAGE DAILY VALUES)  
AND THERMAL POLLUTION,  $Q_p$ , FROM THE NORTHERN STATES  
POWER COMPANY RIVERSIDE AND HIGHBRIDGE PLANTS AND  
THE MINNEAPOLIS-ST. PAUL SANITARY DISTRICT PLANT**

	P	Q <sub>R</sub>	V <sub>a</sub>	U	T <sub>a</sub>	Q <sub>P</sub> (cal sec <sup>-1</sup> × 10 <sup>6</sup> )		
Date	(mb)	(cal cm <sup>-2</sup> day <sup>-1</sup> )	(m sec <sup>-1</sup> )	(%)	(°C)	Riverside	Highbridge	Minn.-St. P.
Jan 1965								
10	996.3	102.	2.2	64.	-17.2	8.70	12.46	7.66
11	986.8	161.	3.1	70.	-15.6	12.79	14.91	9.67
12	991.9	150.	2.7	60.	-20.6	12.04	15.78	9.89
13	994.2	225.	2.2	55.	-25.6	12.55	15.87	9.69
14	995.6	144.	3.0	64.	-21.3	13.52	15.54	9.71
15	998.3	234.	3.4	54.	-16.1	13.22	15.18	9.78
16	1006.1	238.	2.2	68.	-18.3	11.01	13.44	8.14
17	988.5	209.	6.3	74.	-4.4	9.22	10.22	7.76
18	992.9	194.	4.5	57.	-12.2	13.02	14.59	10.06
19	979.0	133.	4.5	77.	-6.1	13.10	15.40	10.30
20	988.2	234.	3.5	79.	-11.1	12.59	14.44	10.22
21	979.0	223.	5.0	77.	-2.8	12.43	14.52	11.26
22	986.1	92.	5.9	76.	-4.4	13.17	14.06	9.86
23	986.1	106.	5.6	80.	-7.8	11.20	12.25	8.66
24	974.6	181.	2.9	86.	-8.3	9.85	11.43	7.61
25	971.6	123.	3.3	93.	-4.4	13.21	14.41	10.50
26	981.7	261.	6.3	75.	-11.1	12.67	14.38	10.13
27	982.4	149.	4.4	68.	-15.0	12.24	15.14	10.51
28	988.5	272.	5.7	51.	-25.6	13.54	15.09	10.09
29	986.1	279.	2.0	57.	-25.6	13.52	14.85	9.68
30	987.8	275.	4.6	54.	-21.7	12.98	14.95	8.27
31	977.0	76.	3.7	69.	-21.1	9.74	13.04	7.06
Feb 1965								
1	983.8	303.	5.2	63.	-22.8	13.80	14.68	9.56
2	990.5	269.	3.7	63.	-25.0	13.73	15.26	9.77
3	995.9	123.	4.7	63.	-21.7	13.51	14.84	9.45
4	996.3	265.	3.8	62.	-20.0	13.45	14.92	9.56
Jan 1966								
5	988.5	99.	3.2	96.	-10.0	11.86	0.	10.67
6	991.2	139.	4.6	60.	-16.7	13.63	13.65	10.73
7	997.3	301.	5.5	56.	-23.9	14.05	14.21	10.09
8	993.6	193.	4.3	63.	-18.3	6.83	15.27	9.26
9	977.0	218.	5.3	65.	-7.2	4.61	12.22	8.51
10	994.6	229.	7.1	54.	-15.0	12.73	14.18	10.82
11	999.0	204.	4.5	54.	-15.0	12.75	14.50	10.71
12	987.8	129.	4.7	83.	-7.2	12.28	14.17	10.62
13	988.8	191.	2.5	85.	-9.4	11.99	13.97	10.79
14	989.2	257.	3.9	65.	-13.5	12.23	14.00	10.49
19	998.3	263.	2.2	67.	-14.4	12.47	12.94	10.37
20	999.7	137.	1.6	72.	-15.0	12.58	13.33	10.48
21	999.3	174.	3.7	68.	-14.4	13.11	12.59	10.34
22	995.6	299.	3.5	60.	-19.4	11.30	10.95	8.55
23	995.3	312.	4.0	56.	-22.8	7.00	10.36	7.59
24	999.1	270.	2.4	52.	-21.7	10.62	13.99	10.13
25	1000.0	243.	1.6	56.	-18.3	13.01	13.07	10.49
26	992.2	168.	5.5	64.	-16.1	13.29	13.46	10.44
27	997.6	297.	7.9	47.	-26.1	13.33	13.67	9.89
28	1001.0	278.	5.7	49.	-29.4	13.73	13.65	9.35
Feb 1966								
18	994.6	393.	5.9	52.	-19.4	11.97	11.10	9.46
19	1006.8	399.	3.8	42.	-20.6	11.35	11.13	8.17
20	1012.2	398.	2.6	46.	-19.4	9.53	9.46	7.15
21	1011.2	410.	3.1	42.	-16.7	12.34	11.22	9.12
22	1010.5	403.	1.8	46.	-12.2	11.66	11.02	9.45
23	1005.1	369.	1.9	63.	-8.9	11.41	11.04	10.03
24	988.5	360.	6.0	62.	-4.4	11.45	11.07	9.75

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<p>An attempt is made to calculate the length of the ice-free reach which develops during the winter below a thermal pollution site on a river. A differential equation for the steady state heat balance of a volume element of a river is developed, which leads to the expression</p> $x = C_x \int_{T_{w0}}^{T_{wx}} \frac{dT_w}{Q^*}$ <p>where x is distance downstream from the pollution site to the cross section where the water temperature equals <math>T_{wx}</math>, <math>T_{w0}</math> is water temperature at x equals zero, <math>Q^*</math> is rate of heat loss from the water surface, and <math>C_x</math> is a constant which includes flow velocity and depth. The value of x at <math>T_{wx}</math> equals <math>0^\circ\text{C}</math> is taken as the length of the ice-free reach. <math>Q^*</math> is the sum of heat losses due to evaporation, convection, long- and short-wave radiation, and other processes, each of which is evaluated by an empirical or theoretical expression. The two principal limitations in accurately calculating downstream temperature changes are related to difficulties in evaluating the degree of lateral mixing in natural rivers and the convective and evaporative heat</p> <p>Abstract cont'd on next page</p>			

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Abstract cont'd losses under unstable atmospheric conditions. Observations of lengths of ice-free reaches on the Mississippi River are in good agreement with the calculated values. Significant portions of the St. Lawrence Seaway can be kept ice-free by the installation of nuclear reactors at appropriate lo- cations. 1						